```
ClearAll[e, basis, sp, normalize]
e0 = 1;
e1 = x;
e2 = x^2;
e3 = x^3;
basis = {e0, e1, e2, e3};
sp[f_, g_] := Integrate[Conjugate[f] * g * Exp[-x], {x, 0, +∞}]
normalize[f_] := f / Sqrt[sp[f, f]]
```

In[204]:=
```
Orthogonalize[basis, sp[#1, #2] &] // Simplify
```

Out[204]= $\left\{1,\ -1+x,\ \frac{1}{2}\left(2-4x+x^2\right),\ \frac{1}{6}\left(-6+18x-9x^2+x^3\right)\right\}$

```
ClearAll[u, ons]
u0 = normalize[e0];
u1 = normalize[e1 - sp[e1, u0] * u0];
u2 = normalize[e2 - sp[e2, u0] * u0 - sp[e2, u1] * u1];
u3 = normalize[e3 - sp[e3, u0] * u0 - sp[e3, u1] * u1 - sp[e3, u2] * u2];
ons = {u0, u1, u2, u3} // Simplify
```

Out[209]= $\left\{1,\ -1+x,\ \frac{1}{2}\left(2-4x+x^2\right),\ \frac{1}{6}\left(-6+18x-9x^2+x^3\right)\right\}$

In[243]:=
```
ClearAll[p, pe, pu]
pe = x^3 + x^2 + x + 1;
p0 = sp[pe, u0];
p1 = sp[pe, u1];
p2 = sp[pe, u2];
p3 = sp[pe, u3];
pu = {p0, p1, p2, p3}
```

Out[249]= {10, 23, 20, 6}

In[250]:=
```
p0 * u0 + p1 * u1 + p2 * u2 + p3 * u3
% == pe
```

Out[250]= $1+x+x^2+x^3$

Out[251]= True

In[252]:=
```
Simplify[ons.pu]
% == pe
```

Out[252]= $1+x+x^2+x^3$

Out[253]= True

Die erhaltenen Polynome des `ons` sind die Laguerre-Polynome. Siehe auch: Skript 5.§7,2.8(c).